Conversations About Place Value: A Survey of Literature Across Three International Research Communities

Pamela Vale
Rhodes University
pamela.vale@ru.ac.za

Lise Westaway
Rhodes University
l.westaway@ru.ac.za

Place value is a foundational competency for primary school mathematics and for this reason we have sought to investigate what the recent and current academic conversations are around this important concept. In this paper we present a survey of literature presented in the Australasian, European and Southern African contexts through a review of purposively selected conference proceedings and journals to establish what the conversations have been about the teaching and learning of place value in these research communities from 2013 to 2022.

An understanding of place value is a foundational competency for primary school mathematics. Lambert and Moeller (2019) maintain that understanding place value is a predictor of success in primary school and for later number competencies. Recognising this importance, this paper asks: what are the conversations about place value teaching and learning in mathematics education research communities? Specifically, we examine research originating from three international communities: Australasia, Europe and Southern Africa. There is clear potential for conversations to happen across these research communities and value to be had in doing so.

Place value understanding develops over a period of time, therefore, it is important for learners to be familiarised with the base-10 decimal system in the early years. The base-10 system refers to the value of each digit as determined by the position within a number. According to Dehaene and Cohen (1999), place value understanding is based on a triple code model, meaning that there are three representations of number: understanding quantities, number words and number symbols. The focus in schooling is often on procedures, such as, the standard algorithm taught by rote (e.g., Graven et al., 2013). The importance of developing a conceptual understanding of place value requires children to be able to construct the algorithm as opposed to being reliant on a taught procedure (e.g., Benton, et al., 2018). Important in constructing algorithm is the need for teaching the place value concept through resources (manipulatives, iconic representations, and digital tools) (e.g., Larkin et al., 2019).

Methodology

This literature survey was done with five purposively selected conference proceedings and journals that were deemed representative of the three academic research communities: Europe, Australasia and Southern Africa. The content of these publications was conceptualised as representative of the conversations about place value teaching and learning in the respective mathematics research communities. For the European context, we included the proceedings of the bi-annual Congress of the European Society for Research in Mathematics Education [CERME]. For the Australasian context we included the proceedings of the annual conference of the Mathematics Education Research Group of Australasia [MERGA] and papers published in the associated *Mathematics Education Research Journal* [MERJ]. Similarly, for the Southern African context, we included the proceedings of the annual conference of the Southern African Association for Research in Mathematics, Science and Technology Education [SAARMSTE] and papers published in the associated *African Journal for Research in Mathematics, Science and Technology Education* [AJRMSTE]. We searched for the term 'place value' in all the above-mentioned sources from 2013-2022 to identify the papers for inclusion. Only full research papers were included. In total, the corpus of papers totalled 158. We further classified the papers into 3 categories: (1) papers that focus explicitly on the teaching and learning of place value; (2) papers in which the focus is not on place

(2023). In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives*. *Proceedings of the 45th annual conference of the Mathematics Education Research Group of Australasia* (pp. 509–516). Newcastle: MERGA.

value, but it is an important concept in the paper and the work does contribute to conversations about the teaching and learning of place value either through its findings, implications or other commentary; (3) papers in which place value is mentioned, but the mention is inconsequential to the findings and it does not contribute to conversations about the teaching and learning of place value. Those classified as (1) or (2) were read in full to determine the contribution made by each paper to conversations about the teaching and learning of place value.

Table 1

Number of Papers Reviewed and Classification of the Papers

Context: Proceedings / Journal	Total	Category (1)	Category (2)	Category (3)
Australasia: MERGA	41	12	6	23
MERJ	32	3	4	25
European: CERME	65	11	11	43
Southern African: SAARMSTE	12	1	2	9
AJRMSTE	8	3	1	4

Findings

In this section, we present an analysis of the papers making substantive reference to place value across the three international mathematics education research communities. We draw out the themes evident across the papers and summarise their main findings. We make reference to category (1) papers only but provide a full list of papers at: http://bit.ly/3JDiIuk.

In total, there were 15 category 1 papers from Australasia between 2013 and 2022. The themes emerging from these papers include assessment of children's place value understanding (5 papers); the use of resources (4); the role of place value understanding in learning algorithms (4) and teachers' pedagogical practices (2). Twelve are focused on children's understanding, 2 on pre-service teachers [PSTs] and one on teachers' pedagogical practices.

Hurst's (2014) study examines how a diagnostic assessment of children's place value understanding can assist pre-service teachers [PSTs] in making appropriate decisions about learning tasks and resources. Through using the assessment, the PSTs identified that children have difficulty reading and writing numbers, specifically when zero is a place holder and also struggled to interpret numbers and their values. The results show that the diagnostic assessment enabled the PSTs to think about place value in a conceptual way. Bicknell and Young-Loveridge's (2015) research focuses on two assessment tasks given to Year 1-3 children. The tasks focused on the placement of numbers on a 0-10 and 0-20 empty number lines. The results show that learners were more accurate in placing numbers on the 0-20 number line and that those with a better place value understanding were better with placing the numbers on the number lines. They suggest that teachers should make explicit the connections between different representations of two digit numbers. The paper by Gervasoni and Peter-Koop (2015) also focuses on assessment of learner knowledge, in a comparative study of the counting and whole number understanding of Grade 2 children in Germany and Australia. Their findings indicate that there was a significant difference in place value understanding at the end of Grade 1, but that the results of the two cohorts were the same after Grade 2. The authors surmise that the curriculum expectations may have some influence on the differences and suggest that more research is required on this as there may be different pedagogical approaches in the two countries. Young-Loveridge and Bicknell (2016) developed a framework to assess 5-7-year olds' place value development. They show that young learners are able to perform early place value tasks. The authors present important implications for teachers and curriculum showing that: (1) learning of the facts 5+5 and 120+10 are important for place value; and that (2) learning how to combine single digit

numbers to multiples of 10; and (3) making explicit links between the digits in two-digit numbers and groups of tens and ones are important. Hurst and Hurrell (2016a) sought to develop a tool, the Multiplicate Thinking Quiz, to assist teachers in assessing mathematical reasoning. They show that students are taught procedures rather than having the opportunity to develop a conceptual understanding. Of relevance to this review is the finding that the notion of 'times bigger' is not understood by most children.

There is also a focus in the Australasian research on the role of place value understanding in the application of algorithms. Hurst and Huntley (2017) explore whether children make the connection between place value partitioning of a number and the distributive property of multiplication. Their analysis shows that children who only demonstrate a partial understanding of place value partitioning do not consistently apply the distributive property in their calculations. They offer several clear teaching implications, including that teaching should focus on establishing a link between the distributive property and place value partitioning. Downton et al. (2020) also focus on multiplication and contrast the use of a place value partitioning method with a truncated strategy of ignoring the zeros and then re-placing them in the final solution. They argue that this truncation strategy is frequently demonstrated in classrooms and reinforced in textbooks, and they show that the children who thought in this way did not understand place value partitioning and were manipulating the zeros without understanding. Jazby and Pearn's (2015) focus is similarly on multiplication algorithms and compare algorithms based on how they work with place value. They present algorithms that "suspend place value" (p. 311) but explain that this requires cognitive work to reinstate place value at the end of the calculation. Their argument is that different algorithms work with place value differently and as a result require different aspects of cognitive work and thus individual children may prefer different algorithms for different reasons related to the mental work involved. Jacobson and Simpson (2019) turn their attention to PST's conceptions of multi-digit numbers in a replication study. The PSTs were tasked with explaining two worked examples making use of the vertical addition and subtraction algorithms. Relevant to this place value review is their finding that PSTs "with less sophisticated conceptions tend to rely on a calculational or algorithmic approach to multidigit addition and subtraction problems and often speak in terms of position rather than value" (p. 86). They offer the implication that place value understanding evident in addition contexts is not necessarily generalised to subtraction contexts and suggest that addition and subtraction should be intermixed in teacher education.

In the research on the use of resources in teaching and learning of place value, Hurst and Hurrell (2016b) used the 'Marvelous Multiplier', a "sliding strip ….to assists students to understand that when numbers are multiplied or divided by a power of ten, all the digits move one place to the left (for multiplication) or one place to the right (for division) for each power of ten" (p.330). Understanding this idea is seen as an indicator of conceptual understanding, while 'adding a zero' is deemed to be an indicator of procedural understanding. The findings indicate that the use of the manipulative helped children's conceptual understanding of multiplicative relations. The research of Gorman and Way (2018) also focuses on the use of a resource to assist learners' mathematical understanding. In their case, the resource is virtual zoomable number line to develop Year 4 learners' understanding of decimal fractions. They argue that the virtual number line provides more opportunities for children to develop an understanding of decimal density than a static number line. Rogers (2021) examined the use of a computer-based Place Value Assessment Tool and the online version with Year 3-6 children. Rogers (2021) noted that while both tools saved time for teachers, they lacked transparency as the "teachers' judgement and involvement in the process was removed" (p.334). The suggestion made is that teachers be supported through professional development to develop their assessment literacy skills to assist them in interpreting the data. Litster et al. (2019) also explore the use of a digital resource, specifically an iPad app allowing virtual manipulation of the Montessori Number Base-10 blocks. The activities focus on grouping to form tens and hundreds.

Their focus is on the affordances that are offered by the app and they compare this with the affordances offered by the corresponding physical manipulative. Findings showed that the prior achievement of the children influenced which affordances they were able to access. They conclude that in selecting and designing virtual manipulative apps, consideration needs to be made of the prior achievement of the children who will be interacting with the app.

The paper by Choy et al. (2022), "contributes to conversations around making a teacher's thinking visible and enhancing a teacher's pedagogical reasoning by exploring the use of pedagogical documentation" over a series of lessons on division. Pedagogical documentation in this research included a single teacher's Padlet (a digital notice board) entries. The findings indicate that the pedagogical documentation made the unseen practices of the division lesson visible (e.g., the teacher's preference for the formal division algorithm and the use of a mnemonic device to teach the algorithm, rather than making a connection between the chunking strategy and the formal algorithm) and enables teachers to learn from their practices and uncover the invisible aspects of their teaching. Nutchey et al. (2016) report on the use of a Reality, Abstraction, Mathematics and Reflection framework to describe children's mathematical reasoning by observing lessons and semi-structured interview with the teachers. Regarding place value teaching, there were limited opportunities for learners to engage with manipulatives or iconic experience (e.g., a place value chart) in grouping and ungrouping tasks involving standard and non-standard partitioning. They argue that resources (manipulatives and iconic) are critical in creating effective learning for students in secondary schools.

In the European research 11 papers were classified as category 1. Ten of these papers focused on the place value understanding of children, and 1 focused on teacher knowledge. Themes emerging across these papers included the role of language (2); the use of resources (6); numeration units in place value understanding (3); and the role of place value understanding in learning algorithms (1). Ten papers are focused on children's understanding of place value, and one is focused on teacher knowledge.

Houdement and Chambris (2013) focus on the teaching and learning of multi-digit numbers and they present a design study in which they aimed to construct a relation between written numbers, numbers units and quantities (the triple code, Dehaene & Cohen, 1999). They include discussion of the different representations of these numbers in the written number (26), number name (twenty-six) and 'numbers-units-number' (2 tens 6 ones) and compare the number names in French and English. They note the 'irregular' number names in French, and the challenges that these number names pose to children learning about place value. Nguyen and Gregoire (2013) also focus on the French language and conducted a study investigating Vietnamese and Belgian (French-speaking) children's performance on place value tasks. They indicate that Vietnamese has a more "transparent name-number system" (p. 1926) and their findings show that the Vietnamese children performed better when the task was related to the number name. Chambris and Tempier (2017) build on this work in relation to large numbers. They argue that a base-1000 approach is useful in teaching large numbers and might contribute to the development of a sense of quantity. The authors make reference to base 10 'numeration units' and base 1000 'numeration units' and explain that an understanding of the relations between units is important. This work is taken further by Coulange and Train (2019) who write of the usefulness of the "discursive register of numeration units in conceptualising decimal numbers" (p. 403). Their findings of an analysis of three classroom episodes include that children found unit-conversions difficult as a deep conceptual understanding is required of the relationship between units, tenths and hundredths etc. There is a continuity evident in the cross-referencing in these papers, showing a coherent strand of conversation on this topic.

Several papers focused on resources. Tsiapou and Nikolantonakis (2013) examined the use of the Chinese abacus with a group of 12-year-olds, showing that the participants did achieve an understanding of place value concepts when using the tool, but struggled to transfer this

understanding to their work in calculations. Jeannotte and Corriveau (2019) explored Grade 3 children's use of base ten blocks and a "homemade abacus" (p. 443), which comprised a colour-coded place value chart with small objects to represent the numbers in each position on the chart, when solving an arithmetic task. They noted that the children had some difficulty in using these manipulatives to solve the task and comment that the teacher's role is important in helping children to operate with manipulatives and not only rely on them to count and represent numbers. In the paper by Morais and Serrazina (2017), several models representing decimal numbers are explored in a teaching experiment. One in particular, the Decimat, is recognised as offering an important part-whole model which can "promote the understanding of partitioning by powers of ten connected with decimal place value" (p. 393).

Three papers focus on virtual manipulatives and representations of place value. Two make use of the Place Value Chart app (Kortenkamp & Ladel, 2013). Behrens (2015) provides a compelling theoretically driven explanation of the potential for this app to foster substantial understanding of the decimal place value system, and this work is referenced in Behrens and Bikner-Ahsbahs (2017) who report on findings from a research project implementing use of this app. In this research, their findings indicate that the actions and gestures of 'dragging' required when using the app "can accumulate more and more aspects of bundling and de-bundling" (p. 2728) which they argue are important place value concepts leading to the development of the concept of decimal fractions. Schulz and Walter (2019) present the *Stellenwerte üben* app and report on the use of this app by primary school children. They show that "the existence of the described mathematical didactic features of virtual representations does not automatically lead to an intended use" (p. 2947) but report that they did nevertheless see evidence of children using the linked representations of place value.

One paper focused on the role of place value understanding in the performance of standard algorithms for the four operations. Zembat et al. (2022) investigated the nature of teachers' knowledge in relation to their articulation of the role of place value in understanding arithmetic operations. They show that "the teachers rely mostly on common content knowledge that has little or no connections to a solid place value understanding" (p. 3735). There is need for further conversations about teachers' understanding of place value.

There are very few place value papers emerging from the Southern African context. There were 4 category 1 papers. The key themes emerging from the Southern African papers were the role of language in developing an understanding of place value (2) and the assessment of learners' place value understanding (2). Two papers focused on children's understanding of place value and one focused on teacher educators.

The research by Hertzog et al. (2017) and Graven et al. (2015) both explore learners' place value competence using different frameworks. Graven et al. (2015) draw on the Learning Framework in Number [LFIN] of Wright et al. (2006), while Hertzog et al. (2017) focus on the Conceptual Understanding of Place Value which they initially developed for learners in Germany. Their findings indicate that most of the Grade 2–4 South African learners could trade tens and ones and work with non-canonical representations provided that they had visual support. Graven (in Graven et al. 2015) used the Conceptual Place Value aspect of the LFIN framework, to assess two learners' understanding of place value as part of a broader study that explored learners' numeracy proficiency and progression. Both papers suggest frameworks that provide a hypothetical trajectory for place value understanding.

The role of language in developing an understanding of the base-10 decimal number system featured in two research studies. Mostert (2019) examined the linguistic features of isiXhosa and English and the affordances that the spoken and written number words offer in learning place value. In contrast to English, isiXhosa is a transparent language. This means that the spoken numbers

correspond with the written numbers. Mostert (2019) argues that teachers need to capitalize on the transparency of isiXhosa to develop their children's knowledge of place value. The second study that focused on language paid attention to teacher educators (Longwe et al., 2022). The research investigated teacher educators' word use when teaching PSTs to develop early year children's knowledge of place value. The findings reveal that 66% of the naming was mathematical and 34% was non-mathematical (everyday terms). In addition, almost two-thirds of the non-mathematical terms included ambiguous pronouns (using 'this', 'that', 'those' to refer to a mathematical object). The research indicated that teacher educators need to pay more attention to the mathematical terms used that relate to place value.

Discussion and Concluding Remarks

The context contributing the most papers to this review was the Australasian context followed closely by the European context and then the Southern African context. Most papers across all contexts are focused on children's understanding of place value, with a secondary focus on teachers and PSTs. There are many 'best practices' proposed across the papers, which points to the need for a systematic review of place value literature to synthesise these ideas.

In the Australasian papers there were a large proportion of papers focused on the assessment of place value understanding (e.g., Hurst, 2014). In addition, there were papers that examined place value understanding in the context of learning and applying algorithms, which is necessary due to the focus in schooling on algorithms (Graven et al., 2013). There is also attention given to the role of resources in the teaching and learning of place value, particularly virtual resources (e.g., Litster et al., 2019). Notable though is the absence of a focus on language in this context, whereas this is present in the European and Southern African work. In the European context, there is a focus evident on the 'triple code' (Dehaene & Cohen, 1999) through the work of Houdement and Chambris (2013), Chambris and Tempier (2017) and Coulange and Train (2019) in which there is exploration of the numeration units as well as exploration of the role of language in developing place value understanding. As with the Australasian research, there is also a focus evident on the role of resources, including virtual resources (e.g., Behrens, 2015) and there is work that addresses the role of place value understanding in learning to understand and apply algorithms (Zembat et al., 2022). The Southern African context offers the smallest number of papers to this review, pointing to a need for growth in this area. Language is included in the conversations in this context, as seen in the work of Mostert (2019). Assessment also appears as a theme in the Southern African work (e.g., Hertzog et al., 2017). The Southern African research does offer one unique contribution, however, in the work of Longwe et al. (2022) which focuses on teacher educators' work with place value. This is a population that is not researched in the other contexts.

As is evident in this review, each research community has overlapping areas of interest but also has absences in their conversations. Language is not explored in the Australasian work reviewed, virtual manipulatives are not explored in the Southern African literature and neither the Australasian nor the European conversations include consideration of teacher educators. In all contexts, the dominant focus is on children's understanding of place value, with proportionally far fewer studies examining teacher, PST and teacher educator knowledge and practices. One of the implications of this research is that there is clear potential for conversations to happen across these research communities and value to be had in doing so. We propose that collaborations involving more than two research communities hold great potential for moving the field forward.

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